Detection and Recovery of Weak Signals

by

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Ang aking taos-pusong pasasalamat ...

Abstract

An efficient noise dithering procedure is demonstrated for measuring weak doublet spectra with a Fourier transform interferometer where the weak interferograms are sampled by a 1-bit analog-to-digital converter. In the absence of noise, no information is obtained regarding the doublet spectrum because the modulation (AC) term $s(x)$ of the interferogram is undetectable which happens when $|s(x)|$ is less than the instrumental detection limit B for all path difference x values. Extensive numerical experiments are carried out to test the recovery of the following interferogram which represents the doublet spectrum: $s(x) = (s_o/2)\exp(-\pi^2 x^2/\beta)[\cos(2\pi f_1 x) + cos(2\pi f_2 x)]$, for different s_o, linewidth factor β and $<f> = (f_1 + f_2)/2$. Even for short observation times (sampling periods) of $s(x)$, the resonant frequencies can be located at high accuracy over a wide range of $<f>$ and β values. Signal-to-noise ratios greater than 50 are also obtained for the power spectra.

Rapid and accurate recovery of the lost high-frequency components in the undersampled representation of a bandlimited signal $s(x)$ is demonstrated using the simplex projection method (SPM). The spectral extrapolation technique is effective if: 1) Fourier spectrum $S(f)$ of $s(x)$ contains features that are exhibited regularly within the signal bandwidth, and 2) Fourier transform $\{S(m)\}$ of the undersampled representation of $s(x)$ contains sufficient information about the said regularities. The SPM is utilized to determine the various features contained in $\{S(m)\}$ and to establish their possible pattern of appearance. The performance of the recovery procedure is tested as a function of the sampling rate. Two test signals with distinctly different Fourier spectrum profiles are considered: 1) Interferogram of a spectral doublet, and 2) Four-point object. In both cases, the bandwidth of $s(x)$ is known a priori and used to determine the number of unknowns to be solved. For an undersampled interferogram that contains only 54% of the energy of the spectral doublet, 42 unknown components have been calculated to decrease the normalized mean-square error of the interpolated signal by 75% relative to the undersampled data. The extrapolation technique is shown to be robust to the presence of additive noise in $S(f)$. The SPM is also demonstrated on the Raman spectrum of CCl_4.

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