

NOISE IN THE DETECTION AND PROCESSING OF WEAK
SIGNALS: TRADEOFFS AND BENEFITS

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ABSTRACT

The effects of noise in the detection and processing of weak signals is discussed in three contexts: 1) noise-aided detection of weak multifrequency signals; 2) in the behavior of the point-spread function (PSF) of the confocal scanning optical microscope (CSOM) under low photon conditions and 3) in the detection statistics of ultrafast laser pulses from parametric down conversion. In these areas, noise is present at different stages in signal generation and detection. Noise can be used to enhance detection by adding noise to the signal and employing an inversion procedure to obtain complete recovery of a weak signal. We also discuss the tradeoffs involved in the resolution of objects in photon-limited imaging conditions for several CSOM configurations where we find that resolution is ultimately limited by signal-to-noise ratio rather than diffraction. Finally, the effects of detector noise and noise from signal generation is discussed in the final section. The characterization of the effects of noise especially in weak signal conditions has not been amply discussed in literature especially its counterintuitive effects, its effect on resolution in optical imaging and on its presence in ultrafast pulse generation and detection. We employ numerical simulations and compare this with known or derived analytic expressions.

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